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Conditionally Heteroskedastic Factor Models: Identification and Instrumental Variables Estimation

Since the seminal papers by Diebold and Nerlove (1989) and Engle (1987), the issue of parsimonious parametrization of time-varying volatility for multivariate time series is still a matter of debate. On the one hand, Engle's model speci.es the time-varying part of the covariance matrix as a function of a few linear combinations of the observed lagged random variables. On the other hand, Diebold and Nerlove's approach of latent factor ARCH model captures the commonality in volatility movements across several time series through the ARCH structure of a few latent factors. Sentana (1998) is right to stress that "there is some confusion in the literature on the similarities and differences" between the two models. He points out two important issues in this respect:

- First, since the conditioning information relevant to define the conditional covariance matrix of a latent factor ARCH model is not reduced to only past values of the observed variables, the Diebold and Nerlove's model can in fact be regarded as a Stochastic Volatility (SV) model (see for example Ghysels, Harvey and Renault (1996)).
- Second, the Engle's factor GARCH model is observationally equivalent (up to conditional second moments) to a whole family of conditionally heteroskedastic models, including latent factor ARCH models `a la Diebold and Nerlove.

In other words, as far as only conditional second moments are concerned, nothing in the data allows one to discriminate between factor GARCH models à la Engle (1987) and a whole class of SV models. Only the dynamic structure of the conditional covariance matrices can be identified, whatever the nature (latent or not) the factors may have.

The main purpose of this paper is to draw all the relevant conclusions of this observational equivalence result, not only in terms of model specification and identification, but also for the corresponding inference methodologies. Insofar as we can prove that only the SV structure of the factors can be identified from the observed dynamics of conditional second order moments, any inference methodology which is based on something else that the corresponding conditional moment restrictions (and a well suited choice of adapted instrumental variables) is valid only thanks to a jointly maintained assumption which might have nothing to do with the phenomenon of interest, that is volatility clustering and commonality in volatility movements across several time series.