Incorporation of Poisson Jumps into a model with nonparametrically estimated diffusion and application on the CE countries' exchange rate

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The models which use solely Brownian motion as an error term are not able to explain certain stylized facts of financial time series, namely fat tails or high skewness. These models, therefore, do not mimic real data and consequently missprice derivative based on these models. The most obvious way how to incorporate fat tails into an assets return distribution is to include a jump component.

One of the first models where Poisson jumps were included is Merton (1973). Jump process has been studied in the literature on term structure of interest rate and also in the foreign exchange literature, e.g. Ball and Roma (1994), Ball and Torous (1983), Jorion (1988) or Bates (1996). One of the first papers dealing with nonparametric estimations of diffusion function of interest rate process is Ait-Sahalia (1996). He estimates the diffusion function nonparametrically while drift is still parametric. Stanton (1997) extends this paper and present methodology for estimation of both drift and diffusion nonparametrically. None of these two papers assume jump component. Bandi and Nguyen (2000) extend methodology event further and provide complete asymptotic theory for nonparametric estimates of drift, diffusion and jump intensity functions. Their paper is based on Johannes (2000) where he justifies the nonparametric extraction of the parameters and functions controlling the arrival of a jump from the estimated infinitesimal conditional moments.

However, there is one important problem connected with otherwise general methodology of Ait-Sahalia, Stanton or Johannes, namely data requirement. Ait-Sahalia (1996) uses 5500 observations (around 20 years), and Johannes (2000) uses more than 8000. Since I want to estimate the model using exchange rate of Central European (CE) country, I have just 2500 daily observations. This specific problem -relative small data sample- is a reason why I develop methodology which produces estimates with lower standard error than know estimation techniques. The CEE currency should benefit from the presence of Poison jump component because the emerging markets exhibit significantly lower liquidity than the advanced ones.

In the proposed model I modify Johannes (2000) methodology of nonparametric estimation of diffusion function of interest rate with Poisson jumps. The prime interest of my paper is modeling of the exchange rates of the Central European countries. Since I will work in exchange rate context, I model drift as a function of instantaneous expected rate of appreciation of the foreign currency which is equal to the interest rate differential. My model for exchange rate originates has following specification:

$d \log S_t \; ? \; ?(b_t) dt \; ? \; ?(b_t) dW_t \; ? \; ? dJ$, prob(dJ=1)=?(b)dt and $? \; ? \; N(0, ?_?^2)$

where {Wt,t=0} is a standard Brownian motion, S is a nominal price of foreign currency in terms of domestic one; b = r –r* is interest rate differential (IRD); µ(b) is parametric mean reverting drift function; and s(b) is nonparametric diffusion function. The jumps arrive with intensity ?(b) which is a function of absolute value of IRD. The jump sizes are assumed to be normally distributed $? \; ? \; N(0, ?_?^2)$. I assume that mean jump size is 0 and thus jumps just add volatility to the currency price process rather than moves the exchange rate. Moreover, specifying the process in logarithms with mean zero jumps ensures that µ(r) retains its interpretation as the local mean of the process.

Using the simulation studies I show that my proposed methodology is superior in terms of error band to the general Johannes (2000) one for the case of linear and constant jump intensity. Further, I calibrate model on two Central European currencies and two EMS currencies. I find that CE currencies exhibit constant jump intensity whereas EMS currencies exhibit linear jump intensity with respect to interest rate differential (IRD). Moreover, the nonparametric estimates of conditional volatility reveal higher sensitivity of volatility on the size of interest rate differential. In particular, volatility of EMS currencies starts to rise when the IRD reaches 5 percentage points. On the other hand, volatility of CE currencies starts to rise when IRD reaches 10 percentage points.

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